Optimal Interpolation and Prediction in Pulsar Timing

Xinping Deng
Advisors:
W. Coles, G. Hobbs, R. N. Manchester, J. H. Zheng

National Space Science Center, Chinese Academy of Sciences
ATNF, CSIRO

Sept. 9, 2013

Who is Xinping Deng?

- New face for most people here.
- Come from National Space Science Center, CAS.
- I am an engineer working on pulsar navigation, not an typical scientist.
- In order to do pulsar navigation more properlly, I have to study pulsar timing.
- 2011.09-2012.09, I worked with George, Dick and Bill in Australia.
- It is the work I did when I started to learn pulsar timing, already published by MNRAS.
- I am nervous because I have not given a talk in English for almost one year.
- Forgive my nervous and my English.

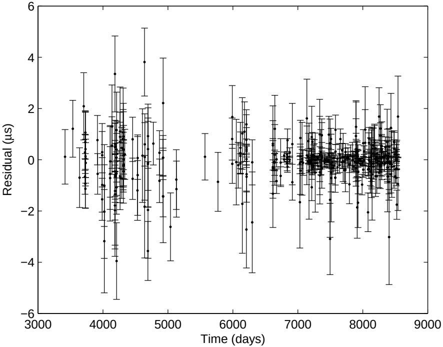
Outline

The talk will be short and easy, NO OUTLINE.

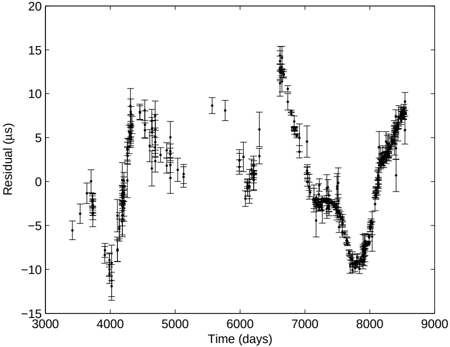
Simulated white and red timing residuals

with real sampling and ToA uncertainties of PSR J1713+0747

- White timing residuals
- ToA uncertainties
- Flat



- Red timing residuals
- ToA uncertainties + Timing noise
- Significant structure



Why we have to model timing noise?

- To determine pulsar **proper motions**.
  - Remove timing noise without affecting any signal with a periodicity close to 1 yr.
- To obtain the characteristic pulse **profile** for a pulsar.
  - It is often necessary to model timing noise if a pulse profile is obtained from a data set that spans many months or years.
- To **fold** pulsar data real-time.
  - The ingored timing noise will hurt real-time pulsar data folding.
- To **navigate spacecraft** in the Solar System.
  - Timing noise prediction can improve the precision of spacecraft navigation.

FITWAVES (×)

FITWAVES is a algorithm to model **timing noise** by fitting a sequence of harmonically related sinusoids to **pulsar timing residuals**.

- It can **not** be extrapolated **past** the end of real observations (or back-extrapolated **before** the start of real observations).
- It requires an **arbitrary** choice of the number of **harmonics** to include in the fitting procedure.
  - **Too few harmonics** implies that **not all the features** in the timing residuals are completely modelled.
  - **Too many harmonics** leads to an **un-physical model** for the timing residuals (particularly when large **gaps** exist in the data).

Maximum likelihood estimator ($\surd$)

- **Given observations:**
  - $o = s + n$
  - o: Timing residuals
  - s: Timing noise
  - n: ToA uncertainties
- **The likelihood function:**
  - $s^{\mathrm{T}}\mathbf{C}_{\mathrm{s}}^{-1}s + n^{\mathrm{T}}\mathbf{C}_{\mathrm{n}}^{-1}n = s^{\mathrm{T}}\mathbf{C}_{\mathrm{s}}^{-1}s + (o - s)^{\mathrm{T}}\mathbf{C}_{\mathrm{n}}^{-1}(o - s)$
- **The gradient of the likehood function with respect to s to zero:**
  - $(\mathbf{C}_{\mathrm{s}}^{-1} + \mathbf{C}_{\mathrm{n}}^{-1})s = \mathbf{C}_{\mathrm{n}}^{-1}o$
  - $\mathbf{C}_{\mathrm{n}}$: Covariance matrix of ToA uncertainties
  - $\mathbf{C}_{\mathrm{s}}$: Covariance matrix of timing noise, can be given by `spectralModel` plugin to `Tempo2`

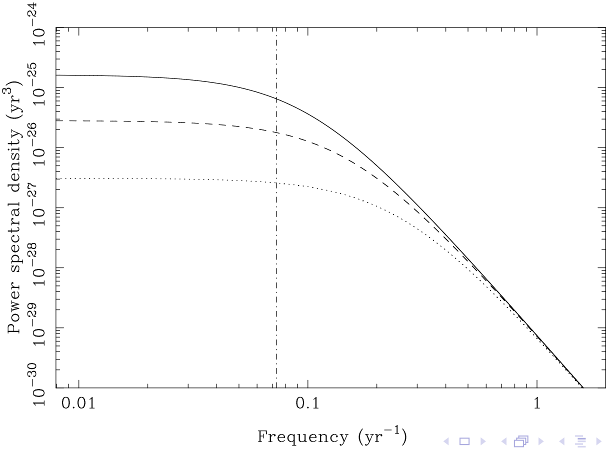
The advantages of the MLE algorithm

- It can deal with **variable sampling error**.
- It can deal with **irregular sampling**.
- It **can** be extrapolated **past** the end of real observations(and back-extrapolated **before** the start of real observations).
- It can give reasonable interpolation in **gaps**.
- It can deal with **weakly non-stationary** observations (e. g. small **glitch** events).

Test the MLE technique by simulations

Spectral densities of simulated timing noise:

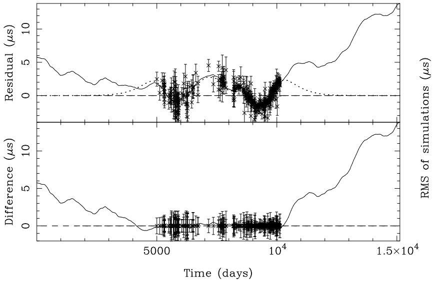
Spectrum model: $P(f) = A/[(f_c/f_o)^2 + (f/f_o)^2]^{(\alpha/2)}$,
$A = 7.6 \times 10^{-30}, \alpha = 4.3$, $f_o = 1\mathrm{yr}^{-1}$, $f_c = 0.1\mathrm{yr}^{-1}$ **(solid)**,
$f_c = 0.15\mathrm{yr}^{-1}$ **(dashed)**, $f_c = 0.25\mathrm{yr}^{-1}$ **(dotted)**.



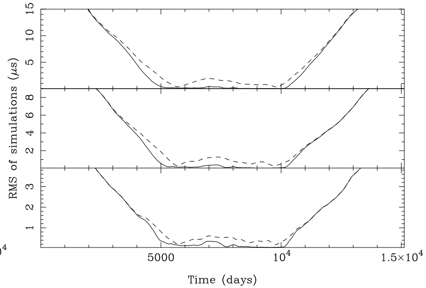
Test the MLE technique by simulations

Simulation result: **(real sampling and ToA uncertainties of PSR J1713+0747 with simulated timing noise)**

One realisation for the dashed spectral density.



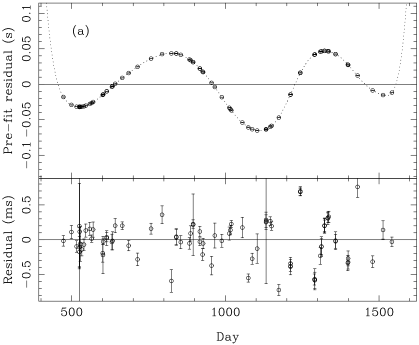
RMS of 50 realisations for each spectral density.



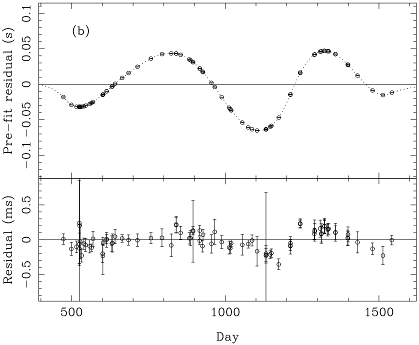
Modelling the timing noise for the Vela pulsar

Note : The lower panel of each figure shows the **difference** between the **real residuals** and the **timing noise model**.

FITWAVES



MLE

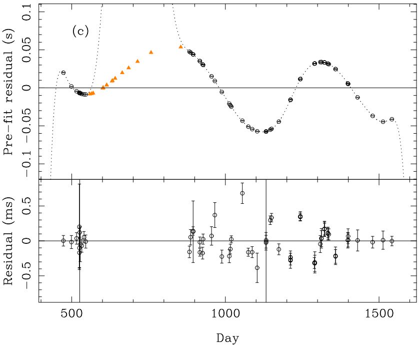


Modelling the timing noise for the Vela pulsar

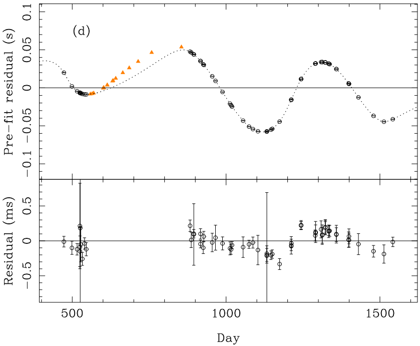
(with **gap** created by deleting observations)

Note : The lower panel for each figure shows the **difference** between the **real residuals** and the **timing noise model**.

FITWAVES



MLE

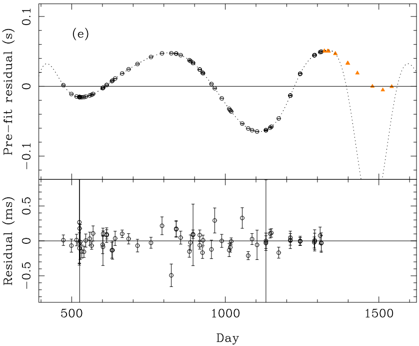


Timing noise prediction for the Vela pulsar

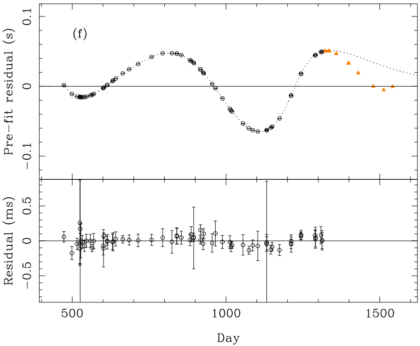
(with 220 days observations deleted)

Note : The lower panel for each figure shows the **difference** between the **real residuals** and the **timing noise model**.

FITWAVES

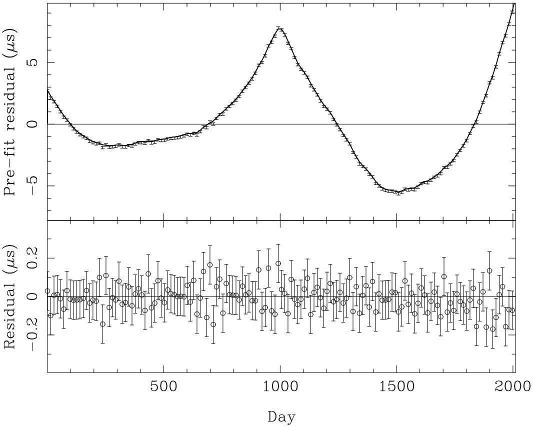


MLE



Modelling simulated timing noise with a glitch event

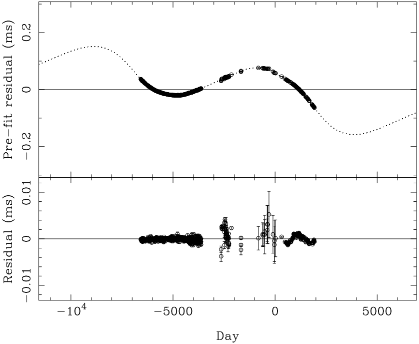
Glitch: Frequency change of 1×10^{-12} Hz.
ToA uncertainties: 100 ns.



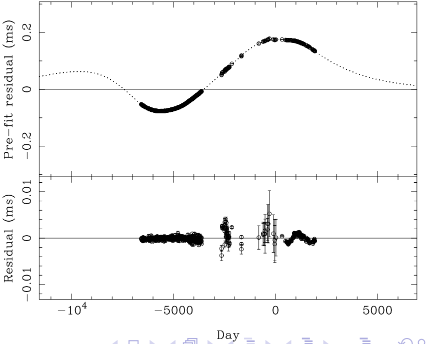
The effect of quadratic removal in the timing noise modeling for PSR J1939+2134

Note : The lower panel for each figure shows the **difference** between the **real residuals** and the **timing noise model**.

Timing noise modeling **without Cholesky** procedure.

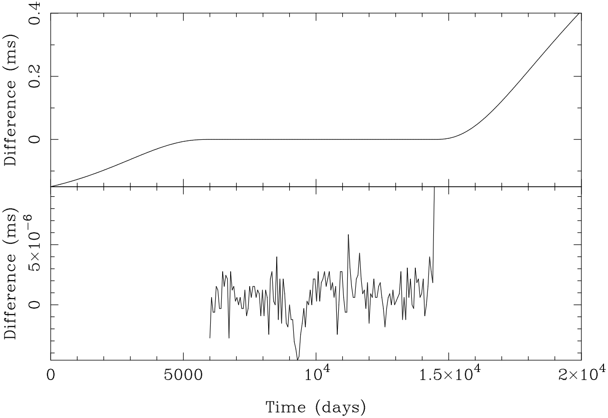


Timing noise modeling **with Cholesky** procedure.



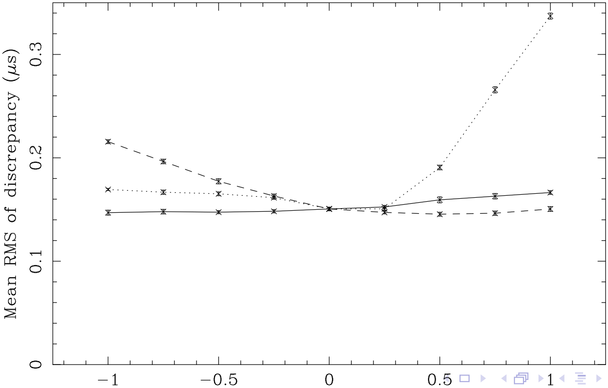
The effect of quadratic removal in the absolute arrival time determination for PSR J1939+2134

Timing model + Timing noise model



Sensitivity to errors in the spectral model of timing noise

Spectrum model: $P(f) = A/[(f_c/f_o)^2 + (f/f_o)^2]^{(\alpha/2)}$
Spectral model error:
$\alpha - \alpha_0$ **(solid line)**, $\log_{10}(f_c/f_{c0})$ **(dotted line)**
ToA uncertainties model error: $\log_{10}(\sigma_w^2/\sigma_{w0}^2)$ **(dashed line)**



Conclusion

We recommend that our new method is **always** used to model, interpolate or extrapolate pulsar timing noise **because**:

- MLE method is applicable even under extreme conditions such as:
  - very steep spectra timing noise;
  - very large gaps;
  - highly variable ToA uncertainties.
- MLE method is close to optimal.

Thank you!